Hence the area of the required triangle is

$$\frac{\triangle\, abc[l(m'n''-m''n')+m(n'l''-l'n'')+n(l'm''-l''m')]^2}{ABC}.$$

Also solved by the Proposer.

315. Proposed by ROBERT E. MORITZ, Ph. D., University of Washington.

Given the area of the segment of a circle of given radius to find the length of the chord.

Solution by G. B. M., ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

I. Let r=radius and $2x$=the length of the chord. Also let A=arc of segment. Then

$$\tfrac{4}{3}x[r-\sqrt{(r^2-x^2)}\,[+\frac{[r-\sqrt{(r^2-x^2)}]^3}{4x}=A.$$

$$\therefore 169x^5+192r^2x^3-168Arx^2+144(A^2-r^4)x-288Ar^3=0.$$

If A and r are known, x can be found.

II. Let θ=angle of segment at center of circle. Then

$$\tfrac{1}{2}r^2(\theta-\sin\theta)=A,\quad x=r\sin\tfrac{1}{2}\theta.$$

By double position θ is found.

III. $r^2[\sin^{-1}\frac{x}{r}-\frac{x}{r^2}\sqrt{(r^2-x^2)}]=A$. Let $\frac{x}{r}=z$.

$\therefore r^2[\sin^{-1}z-z\sqrt{(1-z^2)}]=A$.

$\therefore \frac{2}{3}z^3+\frac{1}{5}z^5+\frac{3}{28}z^7+\frac{5}{72}z^9+\ldots=A/r^2$.

By reversion of series z is found, then $x=rz$.

316. Proposed by J. STEWART GIBSON, Department of Physics, Wadleigh High School, New York City.

Determine the locus of the vertices of parabolas described by particles thrown off from the circumference of a uniformly revolving wheel.

I. Solution by the PROPOSER.

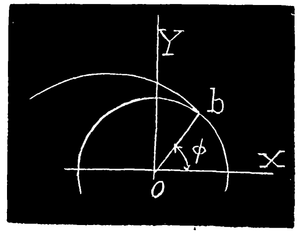
Let r=radius of circle, a=velocity of its periphery, ϕ=angular position of particle b at moment of projection, a_v=vertical component of initial velocity, and a_h=horizontal component of initial velocity. Then $a_v=a\cos\phi$. The height, y_1, to which the particle will rise is (since $h=v^2/2g$),

$$y_1=r\sin\phi+\frac{a^2\cos^2\phi}{2g}. \qquad (1)$$

The time of rise will be $t=\dfrac{a\cos\phi}{g}$. $a_h=a\sin\phi$.

$\therefore x_1$, the abscissa of the vertex of the parabola, is

$$x_1=-ta_h+r\cos\phi=r\cos\phi-\frac{a^2\sin\phi\cos\phi}{g}. \qquad (2)$$



Transposing and squaring (1),

$$r^2\sin^2\phi=y_1^2-\frac{y_1\,a^2\cos^2\phi}{g}+\frac{a^4\cos^4\phi}{4g^2};$$

whence $\cos\phi=\pm\sqrt{-\dfrac{2g(gr^2-y_1a^2}{a^4}\pm\sqrt{r^2-y_1^2+\dfrac{4g^2(gr^2-y_1a^2)^2}{a^8}}}$

and $\sin\phi=\sqrt{(1-\cos^2\phi)}=$

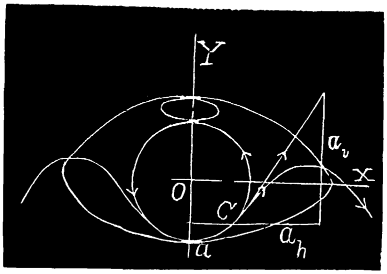
$$\pm\sqrt{1-\left[-\frac{2g(gr^2-y_1a^2)}{a^4}\pm\sqrt{r^2-y_1^2+\frac{4g^2(gr^2-y_1a^2)^2}{a^8}}\right]}.$$

Finally, the equation of the required locus is

$$x_1=\pm r\sqrt{-\frac{2g(gr^2-y_1a^2}{a^4}\pm\sqrt{r^2-y_1^2+\frac{4g^2(gr^2-y_1a^2)^2}{a^8}}}$$

$$\mp\left[\frac{a^2}{g}\sqrt{1-\left(-\frac{2g(gr^2-y_1a^2)}{a^4}\pm\sqrt{r^2-y_1^2+\frac{4g^2(gr^2-y_1a^2)^2}{a^8}}\right)}\right]$$

$$\times\left[\sqrt{-\frac{2g(gr^2-y_1a^2)}{a^4}\pm\sqrt{r^2-y_1^2+\frac{4g^2(gr^2-y_1a^2)^2}{a^8}}}\right].$$



The curve has the following peculiarities: It is symmetrical to the vertical axis; is tangent to the circumference at the inferior apex; and also at superior apex by the inclosed loop. Illustrations of the physical formation of the curve are oil drops from a pulley; mud particles from a carriage wheel; water drops from a revolving grindstone; sparks from revolving fireworks, such as "pin wheels," etc.

II. **Solution by G. W. GREENWOOD, Dunbar, Pa.**

Taking horizontal and vertical axes through the point of projection and in its plane, the path of a particle with initial velocity v, and making initially an angle θ with the vertical, is given by

$$x=v\sin\theta\, t,\ \ y=v\cos\theta\, t-\tfrac{1}{2}gt^2.$$

Eliminating t, we get as the equation to the path described,

$$x^2-\frac{2v^2x\sin\theta\cos\theta}{g}+\frac{2v^2\sin^2\theta y}{g}=0;\ i.\ e.,\ \left[x-\frac{v^2\sin2\theta}{2g}\right]^2+\left[y-\frac{v^2\cos2\theta}{2g}\right]$$

$$=\left[y-\frac{v^2}{2g}\right],$$

which is a parabola whose focus is

$$\frac{v^2\sin2\theta}{2g},\ \frac{v^2\cos2\theta}{2g}.$$

Now taking horizontal and vertical axes through the center of the wheel, and in its plane, the wheel being supposed to revolve clock-wise, the focus of the parabola described by a particle from the point $(-a\cos\theta,\ a\sin\theta)$, a being the radius of the wheel, is given by

$$x=\frac{v^2\sin2\theta}{2g}-a\cos\theta,\ \ y=\frac{v^2\cos2\theta}{2g}+a\sin\theta,$$

which is the equation of the required locus in terms of the parameter θ.

An excellent solution of this problem was received from G. B. M. Zerr.

CALCULUS.

241. **Proposed by C. N. SCHMALL, 89 Columbia Street, New York City.**

Differentiate $y=1+\cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{1+\cfrac{x}{1+\text{etc.}}}}}$

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; FRANCIS RUST, Allegheny, Pa., and the PROPOSER.

The continued fraction is equivalent to $\frac{1}{2}+\sqrt{(\frac{1}{4}+x)}$.

Hence, $y=\frac{1}{2}+\sqrt{(\frac{1}{4}+x)}$, and $\dfrac{dy}{dx}=\dfrac{1}{\sqrt{(1+4x)}}$.

Also solved by G. B. M. Zerr, G. W. Greenwood, and A. H. Holmes.